

# A Highly Sensitive Millimeter Wave Quasi-Optical FM Noise Measurement System

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**Abstract**—A highly sensitive, tunable, low loss quasi-optical millimeter wave FM noise measurement system has been constructed, with state of the art performance. It utilizes a novel matched, easily tuneable quasi-optical cavity in reflection, to act as a carrier suppression filter. This can operate with matched cavity  $Q$ 's of several hundred thousand with almost zero insertion loss to provide an extremely high discriminator slope at low power levels. The FM noise measurement system can allow direct measurement of phase locked sources at low input power levels over ultra-wideband frequency ranges.

## INTRODUCTION

THERE are essentially two approaches to FM noise measurements at high frequencies. The first technique is to phase lock another low noise source in quadrature to the source to be tested. This introduces full FM to AM conversion, but adds the noise of the additional source. The low noise source is commonly a multiple frequency of a very stable reference source. However, at high millimeter wave frequencies the phase noise of these signals can start to be significant, as the phase noise is also multiplied up with the frequency. The other methods are single oscillator techniques, where the carrier is used as the local oscillator and is phase shifted relative to the noise sidebands, using a frequency discriminator. These either involve directly using the slope of a high  $Q$  cavity either in reflection [1] or transmission [2], or a delay line (interferometer) [3], [4], or using a carrier suppression technique (Ondria phase bridge method) [5], [6].

Few  $W$ -band phase noise measurement systems using these techniques have been described in the literature. Ondria [6] has described a carrier suppression system at 94 GHz using a waveguide cavity with a  $Q$  of 6500. Harth [2] has used the slope of a quasi-optical cavity with a  $Q$  of 30 000. However, it was used in transmission which meant the system had an additional 17 dB of insertion loss. Simmons [3] describes a delay line system in waveguide at 80 GHz with an equivalent  $Q$  of around 1500.

Noise measurements become more difficult at  $W$ -band because of the lack of suitable amplifiers at the RF

frequency, and because waveguide and system losses either reduce sensitivity, or require an increase in the amount of input power required to run FM noise measurement systems. Waveguide cavities have  $Q$ 's which are severely limited by resistive losses, and often are not easily tunable. These problems become even more severe at frequencies above  $W$ -band.

One low loss wideband solution is to perform most of the RF signal processing using quasi-optics [7]. Corrugated scalar horns are used to produce highly pure fundamental Gaussian beams which can be manipulated using lenses, mirrors and polarizers.

To achieve the highest sensitivity with FM noise measurements, a key requirement is to have a low loss frequency discriminator with as large a discriminator slope as possible. The measurement technique used in this system employs a very high  $Q$  quasi-optical cavity in reflection to provide an extremely high discriminator slope. Typical working values for the matched loaded cavity  $Q$  are around 100 000 with almost zero insertion loss, although the system can be operated with cavity  $Q$ 's many times larger. This is over an order of magnitude better than that previously achieved with waveguide cavities at this frequency. (An increase in  $Q$  by a factor of 10 can be expected to decrease the noise floor close to the carrier by a factor of 20 dB.) The discriminator slope can be in excess of  $1 \mu\text{V}/\text{Hz}$  for an input power of a few mW. Compared to delay line techniques at this frequency, the improvement in sensitivity can be better than 40 dB. This technique allows very high sensitivity at low input powers over an ultra-wideband frequency range.

## PRINCIPLE OF OPERATION

The system can be thought of as an optical analogue of the carrier suppression noise system used in waveguide [5] but uses a number of novel quasi-optical techniques to effect extremely high performance. Essentially, the signal is split into two, and the carrier is suppressed in one arm using a high  $Q$  reaction cavity, and then reintroduced into the system shifted by 90 degrees. The cavity introduces a frequency dependent phase shift and attenuation across the width of the resonance which has the effect of partial FM to AM conversion. For a perfectly matched cavity,

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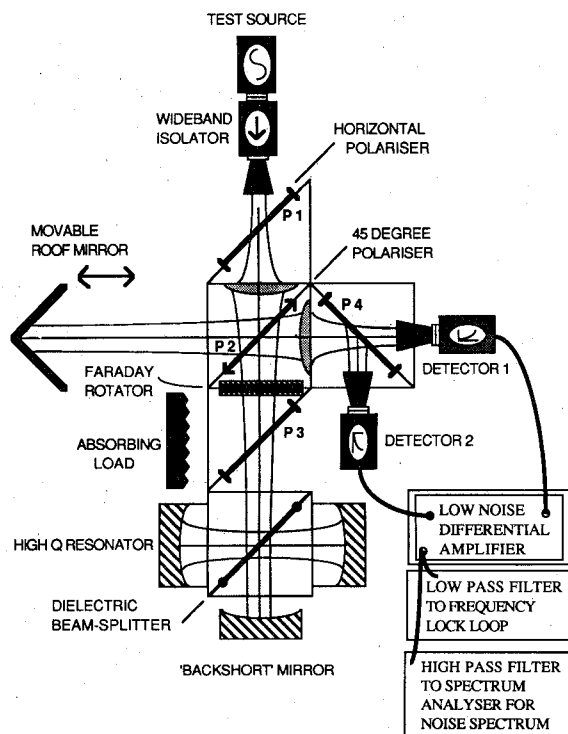


Fig. 1. Schematic diagram of the FM noise measurement system.

full FM to AM conversion is achieved outside the cavity resonance. Balanced mixers are used to distinguish the contributions from FM and AM noise, and to measure the system noise under operating conditions [5].

Fig. 1 shows a schematic of the quasi-optical circuit used in the FM noise measurement system at *W*-band. The oscillator power passes through a wideband waveguide isolator and is then split equally between two arms using polarizer P2. The reflected beam acts as the phase reference or local oscillator arm. The phase of this beam can be adjusted by changing the position of the roof mirror (which flips the polarization through 90 degrees), so that it is in quadrature with the signal returning from the resonator arm. The transmitted beam passes through a 45 degree Faraday rotator (which acts as a quasi-optical circulator) towards the "three mirror resonator." The rotator consists of a permanently magnetized ferrite sheet with quarter wavelength dielectric matching. In conjunction with the polarizers, it produces isolation greater than 30 dB across most of *W*-band, with a VSWR between 1.1–1.4 and an insertion loss of 0.3–0.5 dB [8]. The total isolation between the cavity and the oscillator is believed to be greater than 60 dB which should prevent self injection locking from the high *Q* cavity.

The resonator consists of a near confocal cavity which is coupled via a thin polythene beam-splitter, which is at 45 degrees to the beam and has a power reflectivity of a fraction of a percent. Off resonance, most of the power passes through the beam-splitter and is reflected off the "backshort mirror" and back towards the detection system with almost zero insertion loss. On resonance, power

builds up in the cavity and there is a return signal from the cavity and the "backshort mirror." By altering the phase of the return signal from the "backshort mirror" it is possible to have these two fields cancel at one particular frequency (for an overcoupled cavity). At this frequency, all the input power is absorbed in the cavity and the cavity is thus fully matched. As the absorption losses in the cavity are small, the *Q* of the cavity can be very large and the discriminator slope very steep. In practice, to ensure symmetrical output the "backshort mirror" is positioned an odd number of quarter wavelengths relative to the cavity output. In this case, full matching is achieved only when the one way power loss is equal to the reflectivity of the beam-splitter.

Near resonance, the phase and amplitude of the reflected signal is strongly frequency dependent. Thus, when it combines with the reference signal, the polarization state of the beam emerging from P2 changes rapidly with frequency. This change in polarization state can be detected by splitting the beam equally, with a horizontal/vertical polarizer P4 and detecting the orthogonal plane polarized beam using two nominally identical mixers.

The detectors are both in custom-built tunable cavities which allow excellent matching across *W*-band. In conjunction with the polarizer, the two detectors operate as a quasi-optical analogue of a balanced mixer in a hybrid *T*. The difference signal is amplified using a low noise differential amplifier which allows measurement of the FM noise while discriminating against the AM noise. With careful matching it is possible to obtain about 30 dB of AM rejection across *W*-band.

Because the resonance width of the cavity can be as low as a few hundred kHz, the oscillator frequency can drift off line center during the course of the measurement. It is therefore usually necessary to frequency lock the oscillator to the resonance peak of the cavity, or to lock the resonator to the oscillator frequency. This has been achieved by attaching one of the resonator mirrors to a piezo-electric crystal. It should be noted that this is a frequency lock, as opposed to a phase lock. The dc and low frequency components are used for locking and the high frequency noise is measured on a spectrum analyzer and recorded on a computer.

### THREE MIRROR RESONATOR

Coupling to an open resonator with a beam-splitter at millimeter wavelengths has been described before by French *et al.* [10] for absorption measurements, and Goldsmith [11] for use as a single sideband filter. Both systems used the resonator in transmission rather than in reflection. This technique has the advantage over small hole coupling in that the coupling is relatively constant over a large frequency range, and that the resonator mode can be spatially matched to the propagating Gaussian beam mode. It is also relatively simple to change the beam-splitter within the resonator to provide different coupling at different frequencies.

The "three mirror resonator" is an analogue of many waveguide systems, where full matching to a low impedance device is often achieved by using a series resonance across the waveguide in conjunction with a backshort. We can think of the resonator as an infinite number of simple tank circuits all coupled to the transmission line (free space) via the dielectric beam-splitter. Each of these can be represented as a simple LCR circuit, which is only appreciably excited near resonance. On resonance the load is purely resistive and for small coupling and losses the effective resistance across the transmission line is approximated by

$$R_{\text{eff}} \approx Z_0(\alpha_{\text{resonator}}/R_{\text{dielectric}}) \quad (1)$$

where  $Z_0$  is the impedance of free space,  $\alpha_{\text{resonator}}$  is the fractional one way power loss and  $R_{\text{dielectric}}$  is the fractional power reflectivity of the dielectric beam-splitter.

The technique is effectively matching the impedance of free space to the very small losses in the cavity, and full matching can always be achieved as long as the effective resistance is greater than the impedance of free space. In other words, the power reflectivity of the beam-splitter must be greater than the effective one way power loss in the cavity. (However, the reflectivity of the beam-splitter cannot be too large in comparison to the power loss, otherwise significant distortion of the sidebands can occur). For a symmetrical discriminator function, the backshort mirror is arranged to be an odd number of quarter wavelengths relative to the cavity output. In this case, for low losses and reflectivities, the coupling parameter  $K$  is approximated by

$$K = R_{\text{dielectric}}/\alpha_{\text{resonator}} \quad (2)$$

For optimum operation  $K = 1$  (critical coupling), and the percentage reflectivity of the beam-splitter equals the percentage one way power loss of the cavity.

The power loss in the cavity is made up of absorption losses in the dielectric, absorption losses in the air, diffraction losses and resistive losses in the end mirrors. Experimental values for the fractional loss  $\alpha$  at 94 GHz have been found to be approximately 0.1% for copper mirrors and 0.13% for aluminium mirrors. At  $W$ -band the losses in the air and dielectric are very small and the diffraction losses can be made negligible for the fundamental Gaussian mode, with proper design of the resonator cavity [12]. In practice, the resistive losses in the end mirrors are thought to dominate. For a mirror of conductivity  $\sigma$  at frequency  $f$  the fractional power loss is given by [9]:

$$\alpha_{\text{refl}} = 2(4\pi\epsilon_0 f/\sigma)^{1/2} \quad (3)$$

For copper and aluminium mirrors this leads to respective theoretical absorption losses of around 0.08% and 0.11% for the reflection off one mirror at 94 GHz.

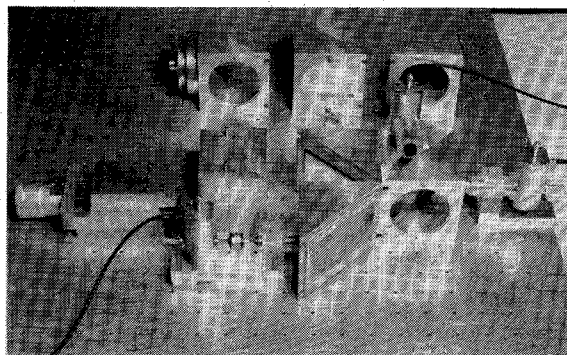


Fig. 2. Illustration showing construction of the FM noise system using half-cubes on an optical breadboard.

The figure of merit for a resonant cavity is the finesse of the system. This is given by

$$F = \pi(1-\alpha)^{1/2}(1-R)^{1/2}/(\alpha+R-\alpha R) \approx \pi/(\alpha+R) \quad (4)$$

where  $\alpha$  is the one way fractional power loss (or round trip amplitude loss) and  $R$  is the fractional reflectivity of the beam-splitter. The maximum finesse occurs in the limit of vanishingly small  $R$  (unloaded cavity). In practice to achieve the maximum discriminator slope the reflectivity of the beam-splitter  $R$  is chosen to approximate  $\alpha$ . This allows full matching to the cavity, leading to a typical operating finesse of around 1500. The reflectivity of the beam-splitter depends on its thickness, refractive index and the polarization state of the incoming beam. In practice, polythene sheets of thickness 10–20  $\mu\text{m}$  have been used successfully as beam-splitters. The polarization state can be flipped simply by turning the circulator around and rotating the polarizer P3 through 90 degrees.

The loaded  $Q$  of the cavity  $Q_L$  is given by

$$Q_L = F \cdot 2L/\lambda \quad (5)$$

where  $L$  is the length of the cavity and  $\lambda$  is the wavelength. Therefore, the  $Q$  simply scales with the length of the cavity, providing that the mirrors are made sufficiently large to keep the diffraction losses negligible for the cavity mode that is excited in the resonator [12].

#### CONSTRUCTION OF SYSTEM

The system has been constructed using a "half-cube" optical breadboard. The half-cubes have a side-length of 120 mm and aperture diameters of 88 mm. High density polyethylene (HDPE) aspherical lenses were used to couple the radiation through the system, and to provide spatial matching to the resonant cavity. These lenses had quarter wavelength blazing to provide low reflection losses in the 90 GHz region. The polarizers were constructed from 25  $\mu\text{m}$  tungsten wire at 50  $\mu\text{m}$  spacing wound on a flat metal frame using a coil winder. At  $W$ -band, the cross-polarization is typically better than 35 dB and the insertion loss is negligible. A photograph of a version of the system is shown in Fig. 2.

## THEORY

This section will consider the theoretical behavior of a carrier suppression system, for a symmetrical cavity response, but where the cavity coupling parameter  $K$  can vary. This is the general case where the backshort mirror is placed an odd number of quarter wavelengths behind the resonator.

In this system, we have one signal split into two paths before being recombined and analyzed with a balanced mixer configuration. Path 1 acts as the phase reference arm and has an adjustable delay. Path 2 contains the carrier suppression filter consisting of a matched or near-matched high  $Q$  reflection cavity where the "backshort mirror" has been adjusted for symmetrical output. The recombined signals are analyzed by a balanced mixer system which discriminates against amplitude noise.

We consider the signal consisting of a carrier at frequency  $\omega_0$  and a small phase noise component at frequency  $\Delta\omega$  away from the carrier. This can be approximated by

$$V = V_0 [\cos(\omega_0 t) - 1/2 \cdot \Delta\phi(\Delta\omega) \cdot \{\sin((\omega_0 + \Delta\omega)t) + \sin((\omega_0 - \Delta\omega)t)\}] \quad (6)$$

where  $\Delta\phi$  is the phase noise modulation assumed to be sufficiently small to produce negligible higher order sidebands.

If we now consider the signal that acts as a phase reference and reflects directly off the roof mirror with a time delay  $\tau_1$  we have

$$V_1 = V_0 \Gamma_1 [\cos(\omega_0(t - \tau_1)) - 1/2 \cdot \Delta\phi(\Delta\omega) \cdot \{\sin((\omega_0 + \Delta\omega)(t - \tau_1)) + \sin((\omega_0 - \Delta\omega)(t - \tau_1))\}] \quad (7)$$

where  $\Gamma_1$  is the frequency independent voltage amplitude transfer function of path 1.

If we now consider path 2 which contains the cavity reflection filter there is now an additional frequency dependent phase and amplitude variation on reflection from the cavity, which is taken as symmetrical. This gives a voltage

$$V_2 = V_0 \Gamma_2 [\Gamma_{\text{cav}}(\omega_0) \cos(\omega_0((t - \tau_2) - \Phi_{\text{cav}}(\omega_0)) - \Gamma_{\text{cav}}(\omega) \cdot 1/2 \cdot \Delta\phi(\Delta\omega) \cdot \{\sin((\omega_0 + \Delta\omega)(t - \tau_2) - \Phi_{\text{cav}}(\omega)) + \sin((\omega_0 - \Delta\omega)(t - \tau_2) - \Phi_{\text{cav}}(\omega))\}] \quad (8)$$

where  $\Gamma_2$  is the voltage amplitude transfer function, and  $\tau_2$  the delay of path 2, without the cavity.  $\Gamma_{\text{cav}}(\omega_0)$  and  $\Phi_{\text{cav}}(\omega_0)$  are the reflection and phase coefficients of the cavity on resonance, and  $\Gamma_{\text{cav}}(\omega)$  and  $\Phi_{\text{cav}}(\omega)$  are the relative frequency dependent amplitude and phase shifts associated with the cavity at a frequency  $\omega = \omega_0 \pm \Delta\omega$ .

For a general cavity coupled to a transmission line with coupling parameter  $K$ , the amplitude reflection coefficient is given by

cient is given by

$$\Gamma_{\text{cav}}(\omega) = \frac{\sqrt{\left[ (K-1) - (K+1) \left( \frac{2Q_L \Delta\omega}{\omega_0} \right)^2 \right]^2 + 4K^2 \left( \frac{2Q_L \Delta\omega}{\omega_0} \right)^2}}{(K+1) \left( 1 + \left( \frac{2Q_L \Delta\omega}{\omega_0} \right)^2 \right)} \quad (9)$$

where  $Q_L$  is the loaded  $Q$  of the cavity. Thus on resonance we have  $\Gamma_{\text{cav}}(\omega_0)$  given by

$$\Gamma_{\text{cav}}(\omega_0) = |K-1|/(K+1). \quad (10)$$

The phase change across the cavity resonance is given by

$$\Phi_{\text{cav}}(\omega) = \tan^{-1} \left( \frac{2K \left( \frac{2Q_L \Delta\omega}{\omega_0} \right)}{(K+1) \left( \frac{2Q_L \Delta\omega}{\omega_0} \right)^2 + (1-K)} \right). \quad (11)$$

It is interesting to note that at the half power points where  $\Delta\omega = \omega_0/2Q_L$  the phase change across the cavity is given by  $\tan^{-1}(K)$  in the general case.

If these two signals appear together at the input of a square law detector the output is approximately given by

$$V_{\text{out}} = C(V_1 + V_2)^2$$

where  $C$  is a constant related to the detector. The detector usually only gives an output at the difference frequency, where the phase detector constant  $K_\phi$  is now given by

$$K_\phi = CV_0^2 \Gamma_1 \Gamma_2.$$

Assuming that only mixing terms between the carrier and the noise terms are significant and just taking terms which relate to the difference frequency.

$$V_{\text{out}}(\Delta\omega) = K_\phi \cdot \Delta\phi(\Delta\omega) \cdot \Gamma_{\text{cav}}(\omega) \cdot 1/2 \cdot \{ [\sin\{(\omega_0 + \Delta\omega)(t - \tau_2) - \Phi_{\text{cav}}(\omega) - \omega_0(t - \tau_1)\} + \sin\{(\omega_0 - \Delta\omega)(t - \tau_2) - \Phi_{\text{cav}}(\omega) - \omega_0(t - \tau_1)\}] + K_\phi \cdot \Delta\phi(\Delta\omega) \cdot \Gamma_{\text{cav}}(\omega_0) \cdot 1/2 \cdot [\sin\{(\omega_0 + \Delta\omega)(t - \tau_1) - \omega_0(t - \tau_2)\} + \sin\{(\omega_0 - \Delta\omega)(t - \tau_1) - \omega_0(t - \tau_2)\}] \}. \quad (12)$$

Now simplifying we have

$$V_{\text{out}}(\Delta\omega) = K_\phi \cdot \Delta\phi(\Delta\omega) \cdot [\Gamma_{\text{cav}}(\omega) \cdot \sin(\omega_0(\tau_1 - \tau_2)) \cdot \cos(\Delta\omega t - \Delta\omega\tau_2 - \Phi_{\text{cav}}(\omega)) - \Gamma_{\text{cav}}(\omega_0) \cdot \sin(\omega_0(\tau_1 - \tau_2)) \cdot \cos(\Delta\omega t - \Delta\omega\tau_1)]. \quad (13)$$

Now we can arrange for  $\omega_0(\tau_1 - \tau_2) = (n+1/2)\pi$ , by adjusting the phase of the reference arm, so the two signals are in phase quadrature. We then have maximum

FM to AM conversion as  $\sin(\omega_0(\tau_1 - \tau_2)) = 1$ . Also subtracting common phase terms and rearranging we have

$$V_{\text{out}}(\Delta\omega) = K_\phi \cdot \Delta\phi(\Delta\omega) \cdot [(\Gamma_{\text{cav}}(\omega) - \Gamma_{\text{cav}}(\omega_0)) \cdot \cos(\Delta\omega t) + \Gamma_{\text{cav}}(\omega_0) \cdot \{\cos(\Delta\omega t) - \cos(\Delta\omega t - \Delta\omega(\tau_1 - \tau_2)) - \Phi_{\text{cav}}(\omega)\}]. \quad (14)$$

The second term in (14) can be rewritten

$$V_{\text{out}}(\Delta\omega) = K_\phi \cdot \Delta\phi(\Delta\omega) \cdot [(\Gamma_{\text{cav}}(\omega) - \Gamma_{\text{cav}}(\omega_0)) \cdot \cos(\Delta\omega t) + 2\Gamma_{\text{cav}}(\omega_0) \cdot \sin((\Delta\omega(\tau_1 - \tau_2) + \Phi_{\text{cav}}(\omega))/2) \cdot \sin(\Delta\omega t - (\Delta\omega(\tau_1 - \tau_2) + \Phi_{\text{cav}}(\omega))/2)]. \quad (15)$$

These two terms have a complex phase relationship with one another, however, combining both terms and calculating  $V_{\text{rms}}^2(\Delta\omega)$  the expression simplifies to

$$V_{\text{rms}}^2(\Delta\omega) = K_\phi^2 \cdot \Delta\phi_{\text{rms}}^2(\Delta\omega) \cdot [(\Gamma_{\text{cav}}(\omega) - \Gamma_{\text{cav}}(\omega_0))^2 + 4 \cdot \Gamma_{\text{cav}}(\omega) \cdot \Gamma_{\text{cav}}(\omega_0) \cdot \sin^2((\Delta\omega(\tau_1 - \tau_2) - \Phi_{\text{cav}}(\omega))/2)]. \quad (16)$$

The first term in the brackets corresponds to FM conversion due to the relative amplitude discrimination with frequency, and the second term corresponds to FM conversion due to the relative phase delay. The amplitude discrimination term is at a maximum for  $K=1$  and becomes less significant for overcoupled and undercoupled cavities. The phase delay term is zero for  $K=1$ , and is small compared to the amplitude discrimination term for undercoupled cavities. However, it can become large for overcoupled cavities for increasing values of  $K$  when the phase change across the cavity approaches 90 degrees. (It is normally arranged that  $\tau_1 \sim \tau_2$  and therefore the  $\Phi_{\text{cav}}$  term dominates close to the carrier unless the coupling is extremely low.)

It should also be noted that for a matched cavity ( $K=1$ ) there is no effect due to any path difference between the two signals, as in this case  $\Gamma_{\text{cav}}(\omega_0)=0$ ,  $K=1$ , and the expression reduces to

$$\Delta V_{\text{rms}}^2(\Delta\omega) = K_\phi^2 \cdot \Delta\phi_{\text{rms}}^2(\Delta\omega) \cdot \Gamma_{\text{cav}}^2(\omega) = K_\phi^2 \cdot \Delta\phi_{\text{rms}}^2(\Delta\omega) \cdot \frac{\left(\frac{2Q_L \Delta\omega}{\omega_0}\right)^2}{1 + \left(\frac{2Q_L \Delta\omega}{\omega_0}\right)^2}. \quad (17)$$

This should be compared to the case where we have a simple delay line, and there is no cavity effect where we have  $\Gamma_{\text{cav}}(\omega_0)=1$ ,  $\Gamma_{\text{cav}}(\omega)=1$  and  $\Gamma_{\text{cav}}(\omega)=0$  for all  $\omega$ .

The output is therefore given by

$$\Delta V_{\text{rms}}^2(\Delta\omega) = 4 \cdot K_\phi^2 \cdot \Delta\phi_{\text{rms}}^2(\Delta\omega) \cdot \sin^2(\Delta\omega(\tau_1 - \tau_2)/2). \quad (18)$$

The sensitivity is now sinusoidal, and the close in sensitivity is far worse than a carrier suppression system for physically realizable systems. A 120 mm cavity resonator with a  $Q$  of 100 000, is roughly equivalent to a loss free delay line of length 100 m at  $W$ -band for the same sensitivity close to the carrier.

The single sideband noise to carrier ratio can be calculated from the relation

$$L(\Delta\phi) = N_{\text{SSB}}(\Delta\omega) (\text{per 1 Hz}) / C = \Delta\phi_{\text{rms}}^2(\Delta\omega) / 2 \quad (19)$$

which is valid for small angle modulation [14]. Here  $N_{\text{SSB}}(\Delta\omega)$  (per 1 Hz) represents the single sideband noise power in a 1 Hz bandwidth at an offset frequency  $\Delta\omega$ , and  $C$  is the carrier power.  $\Delta\phi_{\text{rms}}^2(\Delta\omega)$  is calculated from (16).

The frequency noise can also be calculated, as it is simply given by

$$\Delta f_{\text{rms}}^2(\Delta\omega) = \Delta f^2 \cdot \Delta\phi_{\text{rms}}^2(\Delta\omega) \quad (\text{where } \Delta f = \Delta\omega / 2\pi). \quad (20)$$

Close to the carrier the sensitivity to frequency fluctuations is at a maximum, as the system acts as a frequency discriminator, with a frequency discriminator constant of  $2Q_L/f_0$ , for a matched cavity. However, outside the resonator bandwidth, the sensitivity to frequency fluctuations degrades at 20 dB per decade.

#### SYSTEM NOISE FLOOR

Outside the bandwidth of the resonator, the noise floor of a carrier suppression system is essentially given by the sum of contributions from the two mixers (measured under operating conditions), and the low noise amplifier. Inside the resonator bandwidth the system responds less and less to phase fluctuations but more and more to frequency fluctuations. The relative decrease in sensitivity of the system to phase fluctuations represents an increase in the effective noise floor of the system, and will always degrade at 20 dB per decade close into the carrier. The correction is a function of the coupling parameter  $K$ , and the unloaded  $Q$  of the cavity, and can be calculated from (16). The relative decrease in the noise floor for an unloaded cavity  $Q$  of 200 000 and a frequency of 100 GHz, for overcoupled and undercoupled cavities, is illustrated in Fig. 3. This particularly illustrates the effect of the phase term for overcoupled cavities in the transition region. Fig. 4 shows the system sensitivity to frequency noise, where the response has been normalized to the frequency discriminator response for a matched cavity, which is equal to  $Q_u/f_0 = 2Q_L/f_0$ .

This should be compared with the two oscillator system, where the noise floor is dominated by the phase noise of the multiplied source at millimeter-wave frequencies. It is difficult to make exact comparisons, but in

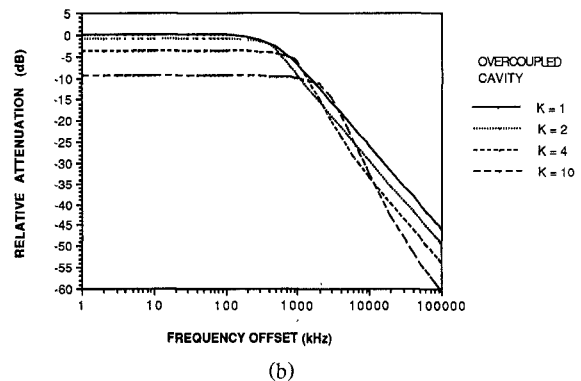
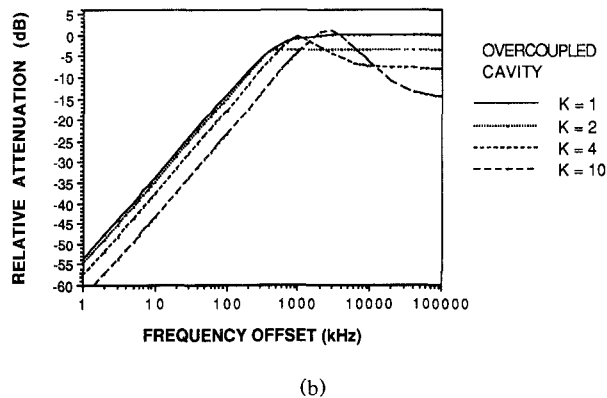
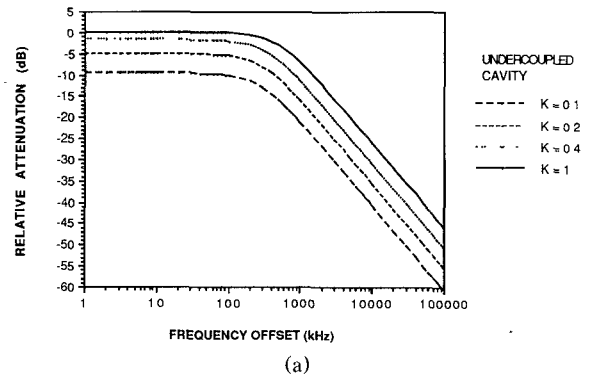
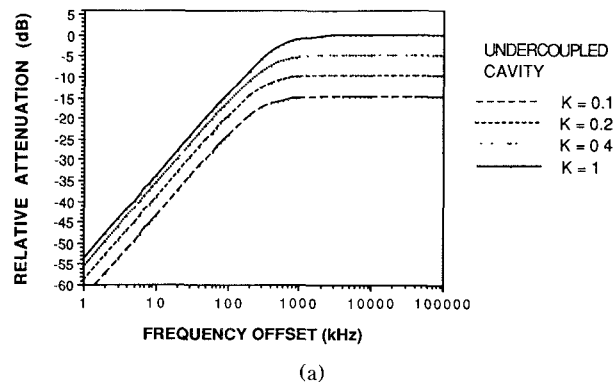


Fig. 3. Graphs illustrating the decrease in system sensitivity to phase noise as a function of offset frequency for overcoupled and undercoupled cavities, for an unloaded cavity  $Q$  of 200 000 at 100 GHz.

Fig. 4. Graphs illustrating the decrease in system sensitivity to frequency noise as a function of offset frequency for overcoupled and undercoupled cavities, for an unloaded cavity  $Q$  of 200 000 at 100 GHz.

general a carrier suppression system with a matched cavity  $Q$  of 100 000 will always have a significantly lower noise floor far from the carrier ( $> 50$  kHz). Closer in to the carrier, the noise floor is roughly comparable to any multiplied source that is capable of wideband coverage at  $W$ -band.

For most free-running Gunn sources at  $W$ -band the measured noise is typically better than 40 dB above the system noise floor. However, it is possible to further increase the sensitivity close in to the carrier of the system, by increasing the  $Q$  of the cavity. This can simply be achieved by lengthening the cavity, or reducing the losses by moving to superconducting mirrors. Lengthening the cavity by a factor of 10 will reduce the noise floor close in to the carrier by 20 dB.

#### CALIBRATION OF SYSTEM

At the present time the system is calibrated using (16). In this case, the system is adjusted for symmetrical response, and the loaded  $Q$  of the cavity is measured by sweeping a calibrated oscillator through the cavity resonance. The loaded cavity  $Q$  is also measured for the other orthogonal polarization state. In this case, the losses remain the same, but the reflectivity of the beam-splitter changes by a known amount (to a good approximation). The one way power loss of the resonator and the reflectiv-

ity of the beam-splitter can be calculated with reference to (4) and (5), from which the coupling parameter  $K$  can then be found using (2).

The noise plots obtained using this calibration have proven very repeatable and consistent, for different values of coupling parameter  $K$ , loaded  $Q$  and oscillator power levels. The results have also been consistent with measurements made using other measurement techniques.

A more ideal calibration would be one where the response of the system could be measured for a signal with a known frequency modulation at a given offset frequency. One possible approach is to use a bridge method where the signal is split, and one arm is amplitude modulated and then recombined 90 degrees out of phase with the signal from the other arm to produce a known phase modulation. Another approach is to use a calibrated reference signal or noise source, to create a known amount of FM and AM modulation. Another technique is to frequency modulate with sufficient magnitude, such that all the power in the carrier is transferred to the sidebands, when the magnitude of the first sideband is theoretically defined (assuming the large signal frequency modulation of the source is linear). However, all these techniques become more difficult to achieve reliably over large offset ranges and to a high degree of accuracy at  $W$ -band and above.

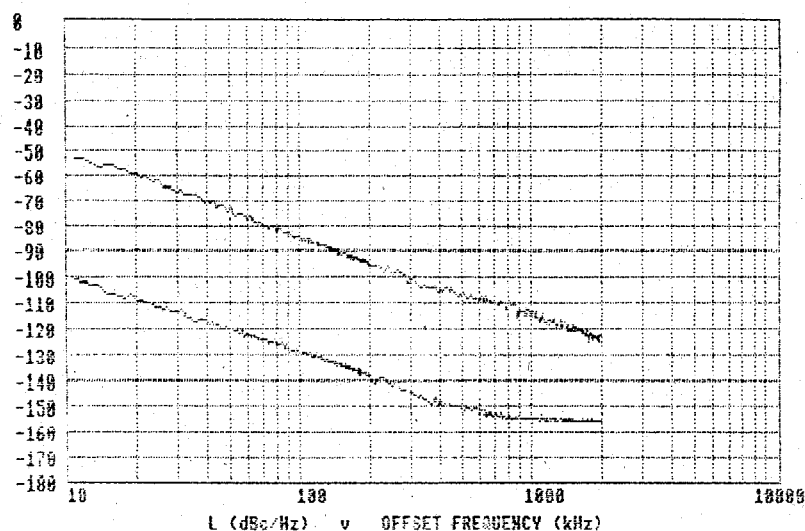


Fig. 5. Typical FM noise measurement of a free-running GaAs second harmonic Gunn oscillator operating in a resonant cap cavity at 88 GHz. The system noise floor for this measurement is also indicated.

#### PERFORMANCE OF FM MEASUREMENT SYSTEM

The system has been used routinely to measure the free-running FM noise characteristics of many different types of diode at  $W$ -band under different cavity and bias conditions. It has been tested at different levels of signal attenuation and cavity  $Q$  and has shown excellent repeatability and consistency. Because it is a homodyne system it is also free from any spurious noise components caused by extraneous mixing products sometimes associated with two oscillator techniques.

The sensitivity of the system with a matched cavity  $Q$  of 100 000 allows FM noise measurements of typical free-running Gunn oscillators at  $W$ -band at the 50  $\mu$ W power level. 1 mW is usually sufficient to make measurements on phase locked and cavity stabilized oscillators at  $W$ -band. A typical noise measurement of a free-running Gunn oscillator at 88 GHz is illustrated in Fig. 5, along with the effective system noise floor for the measurement. It should be noted that this noise floor includes the correction to the system sensitivity illustrated in Fig. 3. The offset frequency in this measurement was limited at 2 MHz only by the low noise amplifiers that were used, and close in by the spectrum analyzer. Measurements have also been performed at 140 GHz at power levels of only several hundred microwatts.

It is hoped that this technique will allow direct measurement of the FM noise of phase locked and cavity stabilized solid-state sources at the higher millimeter wave and sub-millimeter wave frequencies, where high power levels are not always available.

#### CONCLUSION

A quasi-optical FM noise measurement system has been constructed, that uses a novel three mirror resonator system, to provide an extremely steep frequency discriminator slope. Coupling into the resonator via a low reflectivity beam-splitter permits the system to be wide-band

tunable, and overcomes many of the problems associated with small hole coupling into open resonators (where the coupling is a very strong function of frequency). In conjunction with the quasi-optical circulator we believe that this type of resonator represents an advance in state of the art performance, and has many other potential uses such as the measurement of loss and permittivity in dielectrics, sideband filters, scanning interferometry at high millimeter wave frequencies and cavity injection locking of oscillators. Some of the above applications have also been successfully applied at St. Andrews.

As a FM noise measurement system at  $W$ -band, we believe that in terms of the combination of very wide tunability, low loss, and high sensitivity it represents an advance in state of the art performance at high millimeter wave frequencies, at a comparatively low cost. Moreover, these advantages become even more apparent at frequencies higher than  $W$ -band.

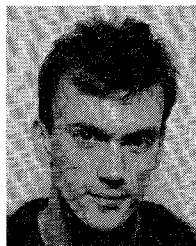
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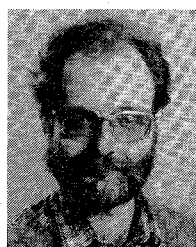
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